<u>Truss Calculations: A How To….</u>

<u>Purpose</u>: Truss calculations are important in determining the maximum forces acting on an object, including bridges, beams and other static objects. These calculations allow engineers to determine the proper materials to build the object, while also giving the architects a means of reducing cost my minimizing excess materials.

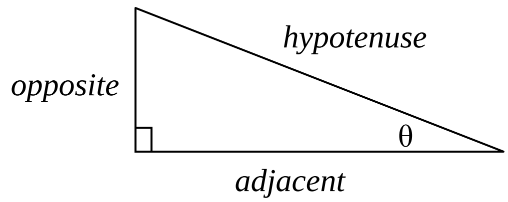
<u>Things to know:</u>

1. **<u>Trigonometry</u>**: Knowing the proper times to use the trigonometric functions are essential. The functions are listed below.

   a. $\sin\theta = \dfrac{opposite}{hypotenuse}$

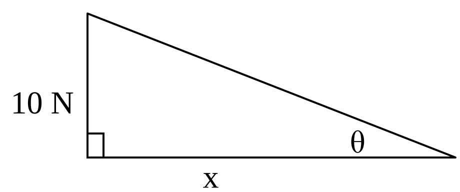
   b. $\cos\theta = \dfrac{adjacent}{hypotenuse}$

   c. $\tan\theta = \dfrac{opposite}{adjacent}$



By definition, the **hypotenuse** is always the longest side (opposite the 90˚), the **opposite** is the side opposite (not touching) the angle θ, and the **adjacent** is the side next (touching, but not the hypotenuse) to the angle θ.

To solve for sides: When **given one side of a right triangle and the angle θ**, it is possible to **solve for any other side** of the given triangle. For example, if a triangle has an opposite side length of 10 Newtons and has an angle of 30˚ as seen in the triangle below, you can solve for either the adjacent or the hypotenuse.



To solve for the **hypotenuse**…

Substitute Known Values

$$\sin\theta = \frac{opp}{hyp} \quad \Rightarrow \quad \sin 30° = \frac{10N}{hyp}$$

Multiply both sides by the hypotenuse

$$hyp \times \sin 30° = \frac{10\text{N}}{\cancel{hyp}} \times \cancel{hyp}$$

$$hyp \times \sin 30° = 10N$$

Divide both sides by the sin of 30˚ to find the hypotenuse

$$hyp \times \frac{\cancel{\sin 30°}}{\cancel{\sin 30°}} = \frac{10N}{\sin 30°}$$

Solve for the hypotenuse.

$$hyp = \frac{10N}{\sin 30°} = \frac{10N}{0.5} = 20N$$

To find the **adjacent** side, you can…

$$\cos\theta = \frac{adj}{hyp} \quad \Rightarrow \quad \cos 30° = \frac{x}{20N}$$

Substitute known values

$$20N \times \tan 30° = \frac{x}{\cancel{20N}} \times \cancel{20N}$$

Multiply both sides by 20 N

$$20N \times \tan 30° = x = 11.55N$$

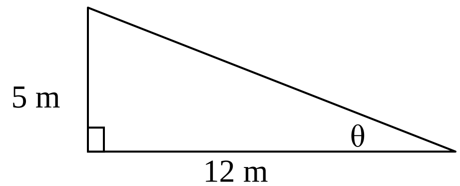
Solve for x

2. **More Trigonometry** – Finding angles and using inverse trigonometry.

   a. Many times, you will be able to measure the sides of a triangle, but unable to measure the angle. When **given two sides of any right triangle**, you can **find the angle** using inverse trigonometry.

   For example, when the two sides of the triangle are given to be 5 meters and 12 meters, the angle can be found using the inverse tangent ($\tan^{-1}$) function.

   Solve for θ by…



Substitute for known values

$$\tan\theta = \frac{opp}{adj} \quad \Rightarrow \quad \tan\theta = \frac{5m}{12m}$$

Take the inverse tangent of both sides

$$\tan^{-1} \times \tan\theta = \tan^{-1}\left(\frac{5}{12}\right)$$

$\tan^{-1} \times \tan = 1$

Solve for θ

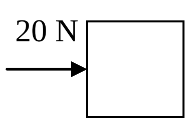
$$\theta = \tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}(.41667) = 22.62^\circ$$

$$\theta = 22.62^\circ$$

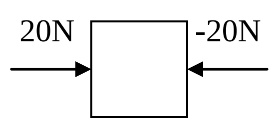
*** This process can be done with* ***$\sin^{-1}$*** *and* ***$\cos^{-1}$*** *depending on the sides given. ***

3. **Vectors**: A deep understanding of vectors is essential in properly solving for trusses. A **vector** by definition is a **value that has a direction**. Vectors can represent forces, velocity, distances, moments, magnetic and electric fields and many others.

   Vectors are unique in that they are **independent of axis**. This means only a vector acting on the same axis may affect another vector on the same axis. For example, if a box is pushed with a force of 20 N to the right on the x-axis, only another force on the x-axis may change the 20 N force. Another force of 20 N to the left will stop the 20 N

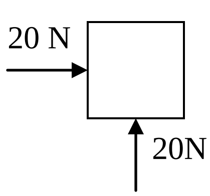


The net force = 20 N right

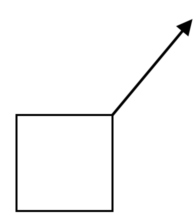


The net force = 0 N

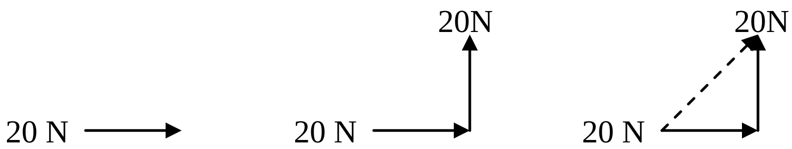
   However, if, instead of a force to the left, another force of 20N is applied in the upward direction, then the force on the box is going both up and right.



The net force on the box would be 28.28N in the up-right direction



To solve for the net force in the above box, use the x and y coordinate system to view the vectors geometrically. The first force is pushing right at 20 N. Next, the second force is pushing up. Using the head-to-tail method, we add these vectors. The sum of the vectors is represented by th2e dotted arrow.



$$A^2 + B^2 = C^2$$
$$20^2 + 20^2 = C^2$$
$$400 + 400 = C^2$$
$$800 = C^2$$
$$\sqrt{800} = \sqrt{C^2}$$
$$28.28N = C$$

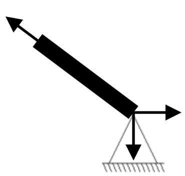
Then, to find the resultant vector (the sum of both (all) vectors), use Pythagorean's Theorem…

4. **<u>Forces</u>:** A force is a push or a pull on an object. There are many different types of forces and infinite ways forces can act and be produced. However, concerning trusses, there are a few that are extremely important.

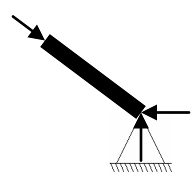
   a. **External force:** These are ALL the forces acting on the entire truss, including weights, reaction forces and other forces acting directly on the truss

   b. **Reaction force:** Forces acting on the truss due to the reaction force of a pin or roller, normal force, and any other object *reacting* on the truss according to Newton's Laws of Motion: For every action, there is an equal and opposite reaction.

      i. **Pins**: Pins are used to connect joints. One example of a pin is a bolt connecting a beam to a bridge. **Pins** hold the joint **in two dimensions**, so there are two reaction forces holding the pin: one in the x-direction; one in the y-direction.



As the beam is pulled up and left, the pin reacts and pulls down and back to the right.



As the beam is pushed down and left, the pin reacts and pushes up and back to the left.

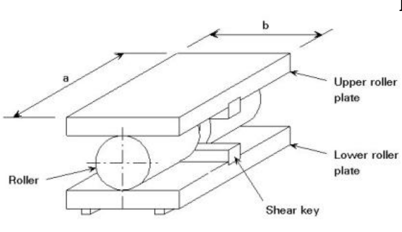
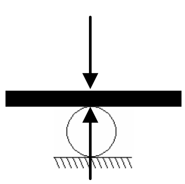
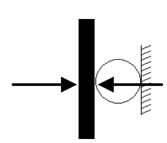


Figure 5 Roller bearing.

      ii. **Rollers:** Rollers are designed to allow movement in one direction and support in another direction. Unlike the pin connection, there is only one reaction force at a pin.

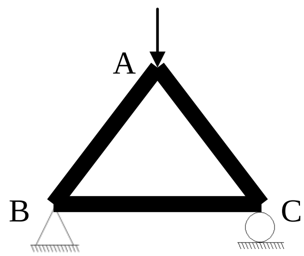


As the beam pushes down, the roller reacts and pushes back up on the beam. The roller does not support left or right



As the beam is pushed right, the roller reacts and pushes left on the beam. The roller does not support up or down

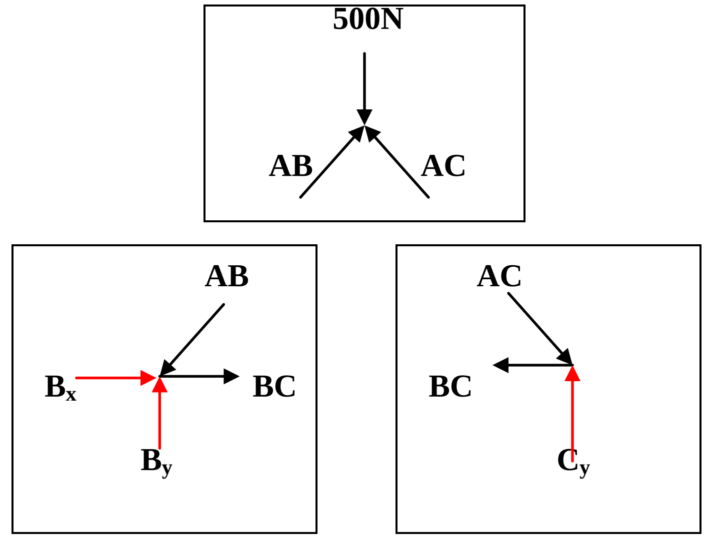
c. **Internal force:** These forces act only with-in the beams of the truss, and usually only include the tension and compression forces. When considering internal forces, create free-body diagrams at specific points where the beams connect. When solving for the internal forces, the **external forces** must also be considered, as they **are responsible for the internal forces**.



Internal Forces:
AB, AC, BC

Reaction Forces:
B_x, B_y, C_y

Free Body Diagrams:

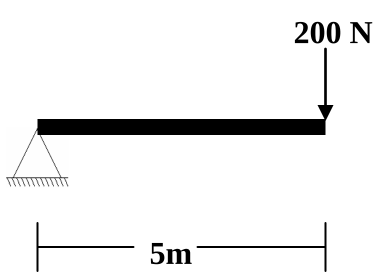


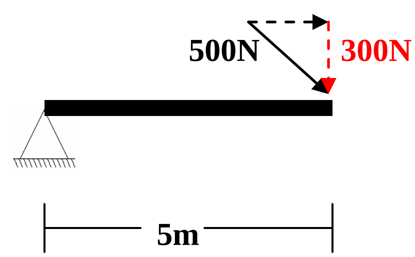
For this truss, there are three free body diagrams that need to be drawn to show the internal forces. Notice that the reaction forces are also drawn, as they are responsible for the internal forces.

5. **Direction of Forces:** To determine the direction of the forces, try to determine the direction the forces would be pushing. For example, since AB is pushing down a left, it is possible that the reaction force, B_x, is pushing right to support the load. However, to determine the actual value, the truss must be calculated. If B_x is shown to be **negative, the force is acting in the opposite direction**. However, once you have decided upon a direction of a force, do not change it – **keep the estimated directions throughout the calculation**. Use the negative forces throughout the rest of the calculation.

6. **Moments:** Moment (Torque) is the tendency of an object to rotate around an object. To calculate the moment of an object, you must have a **perpendicular force** acting on an arm at some radius to the point of rotation

M = F*d
M = 5m * 200N
M = 1000 Nm
Clockwise

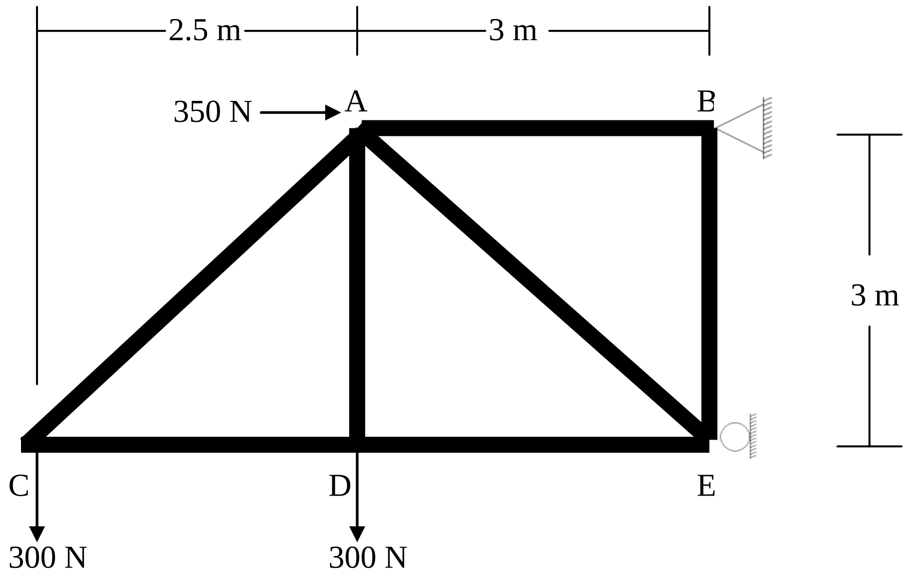




You must consider the perpendicular force when calculating moments.

M = F*d
M = 300N * 5m
M = 1500 Nm

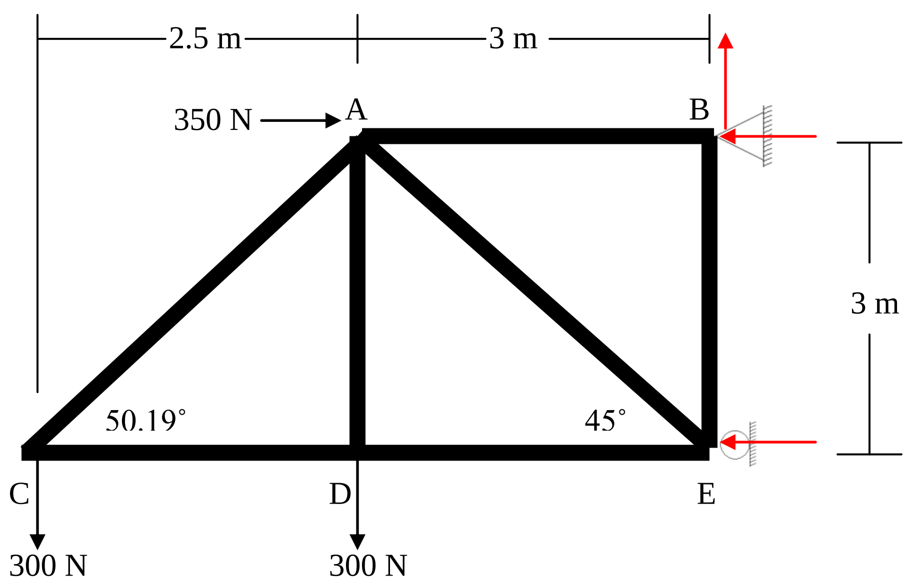
7. Calculate the forces in the truss below. Use the values below



Step 1: Determine where all of the force are acting in the truss

External Forces	Internal Forces
- A_x = 350 N	AB = ?
- B_x = ?	AC = ?
- B_y = ?	AD = ?
- C_y = - 300 N	AE = ?
- D_y = - 300 N	BE = ?
- E_x = ?	CD = ?
	DE = ?

External Forces: B_x, B_y, and E_x are reaction forces due to the pin and the roller. Before continuing, however the direction of the force need to be determined



Since the 350N force is pushing right, the reaction force B_x is most likely pushing left. Also, since the majority of the forces in the truss are pulling down, it is probable that B_y is holding the truss up.

As the truss rotates around the pin, it will force the truss into point E. The reaction force at point E must then push out to support the truss

Step 2: Look at the External Forces only. For the truss to be in equilibrium, the sum of all the forces and the sum of the moments must equal zero for the external forces. Solve for any unknown values.

$\sum F_x = 0$ — Sum of the forces in the x-direction must equal zero.

There are only three: The reaction forces of the pin and roller B_x and E_x, and the 350N .

$A_x - B_x - E_x = 0$

Since the 350N is right, it is positive; the other two are negative because they are left.

$350N - B_x - E_x = 0$ — Cannot be solved….YET! Go onto $\sum F_y$

$\sum F_y = 0$ — Sum of the forces in the y-direction must equal zero.

There are three: B_y, which is the reaction force of the pin, and C_y and D_y, which are external forces.

$B_y - C_y - D_y = 0$

Solve for B_y

$B_y - 300N - 300N = 0$

$B_y - 600N = 0$

$B_y = 600N$ — The reaction force B_y is 600N in the upward direction

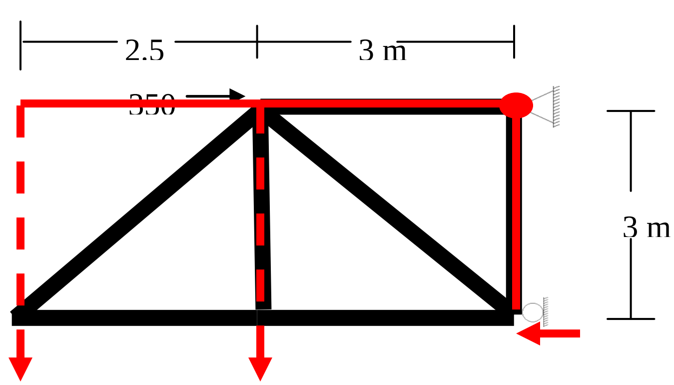
Go to $\sum M$

The sum of the moments about ANY point on the truss must equal zero. Therefore, try to **find the most advantageous point** and take the moments about that point. Hint: Take the moment about the point where most of the unknown forces go through; this will set the moments of the forces to zero at this point as there is no moment arm.

Take the moments around Point B as shown. This gets rid of the unknown forces at this point. The forces at Points C, D and E all create moments about point B.

Since both forces at points C and D are vertical, we consider the distance of the moments from the right, as this is the perpendicular distance, giving a radius of 5.5 meters for point C, and a radius of 3 meters for point D.

Since the Force at point E is horizontal, the perpendicular distance is 3 meters.



$$\sum M = 0$$

The sum of the moments equals zero

$$M_C + M_D - M_E = 0$$

M_C and M_D act counter clockwise, and therefore are positive, M_E acts clockwise, and therefore is negative.

$$(F_C)d + (F_D)d - (F_E)d = 0$$

The Moments are equal to the perpendicular Force acting on the object multiplied by the distance from the point of rotation.

$$(300N)(5.5m) + (300N)(3m) - E_x(3m) = 0$$

Substitute the known values; Solve for E_X

$$1650Nm + 900Nm - E_x(3m) = 0$$

$$2550Nm - E_x(3m) = 0$$

$$2550Nm = E_x(3m)$$

Finish solving for E_X

$$850N = E_x$$

The Force at E_X equals 850 N.

Given that E_x = 850 N, Plug this back into the Sum of Forces Equation.

$$\sum F_x = 0$$

$$A_x - B_x - E_x = 0$$

$$350N - B_x - E_x = 0$$

From above

$$350N - B_x - 850N = 0$$

Substitute in E_x = 850N into the equation

$$-B_x - 500N = 0$$

Solve for B_x

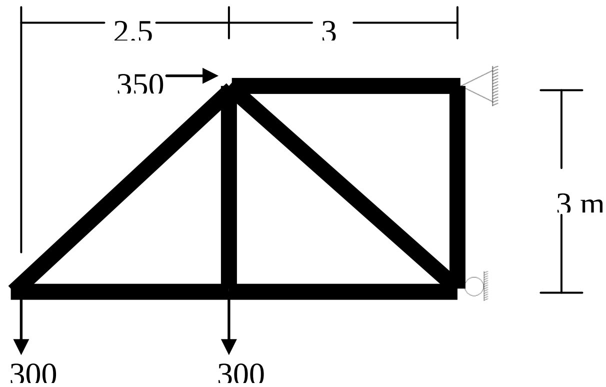
$$B_x = -500N$$

Since B_x is negative, it is in the OPPOSITE direction of what was estimated. Since we estimated that the reaction force of the pin was pushing left, then the reaction force is really pulling right on the truss.

Now that all of the external forces have been solved for, the internal forces need to be solved for. To do this, you must look at each point at which they react. Start with points that have the **least amount of unknown values** .

Tension or Compression: First, determine whether each point is at Tension or compression. To do this, try to figure out how each of the forces react with one another and use your best guess

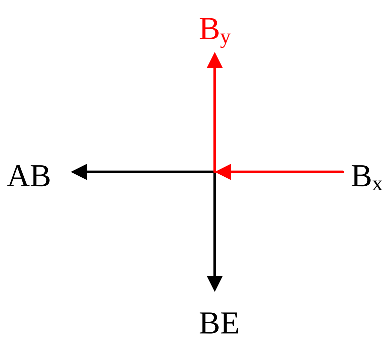
AB = Tension
AC = Tension
AD = Tension
AE = Compression
BE = Tension
CD = Compression
DE = Compression



To draw free-body diagrams with each, a **tension forces pulls out from the point**, and a **compression force pushes into a point**.

Draw the free-body diagrams and solve for the internal forces at each point.

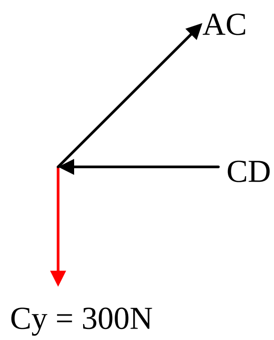
Point B: Point B has two unknown values in both the x and y direction; However, the reaction forces on the pin has to equal the forces in the truss.



$$\sum F_x = 0$$
$$-AB - B_x = 0$$
$$AB = -B_x$$
$$AB = -(-500N)$$
$$AB = 500N$$

$$\sum F_y = 0$$
$$B_y - BE = 0$$
$$B_y = BE$$
$$BE = 600N$$

Point C: Point C only has 2 unknown values, and can be solved with the given information.

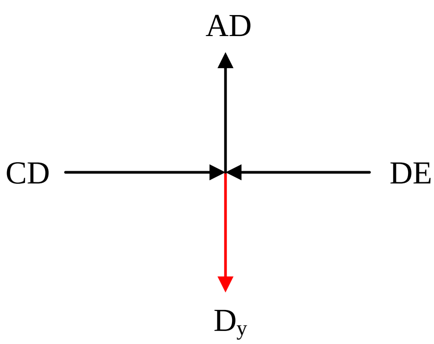


$$\sum Fx = 0$$
$$ACx - CD = 0$$
$$CD = ACx$$
$$CD = AC(\cos 50.19°)$$
$$CD = 390.54N(\cos 50.19)$$
$$CD = 250N$$

AC_y = AC sin 50.19°
AC_x = AC cos 50.19°

$$\sum Fy = 0$$
$$ACy - 300N = 0$$
$$ACy = 300N$$
$$AC \sin 50.19° = 300N$$
$$AC = \frac{300N}{\sin 50.19}$$
$$AC = 390.54N$$

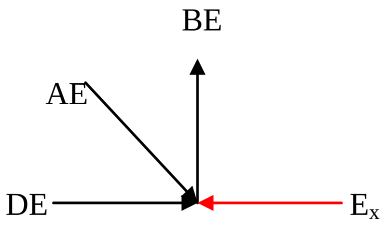
Point D: Now that Point C has been calculated, look at point D since it is connect.



$$\sum Fx = 0$$
$$CD - DE = 0$$
$$DE = CD$$
$$DE = 250N$$

$$\sum Fy = 0$$
$$AD - Dy = 0$$
$$AD = Dy$$
$$AD = 300N$$

Point E: Now that all members have been solved for except AE, we'll look at point E.



$AE_x = AE \cos 45^\circ$
$AE_y = AE \sin 45^\circ$

$$\sum Fx = 0$$
$$AEx + DE - Ex = 0$$
$$AEx + 250N - 850N = 0$$
$$AEx - 600N = 0$$
$$AEx = 600N$$
$$AE \cos 45° = 600N$$
$$AE = \frac{600N}{\cos 45°}$$
$$AE = 848.5N$$

In Summary,

<u>External Forces</u>

- A_x = 350 N Right
- B_x = 500 N Right
- B_y = 600 N Up
- C_y = 300 N Down
- D_y = 300 N Down
- E_x = 850N Left

Internal Forces

AB = 500 N Tension
AC = 390.54 N Tension
AD = 300 N Tension
AE = 848.5 N Compression
BE = 600 N Tension
CD= 250 N Compression
DE = 250 N Compression